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Contact Problems of the Theory of Roller Squeezing of Leather

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Abstract. In the process of roller pressing of a leather semi-finished product, two phenomena occur simultaneously - contact interaction and moisture filtration. This, in turn, requires the joint solution of two types of problems - contact problems and hydraulic problems. The main contact problem of the theory of roller pressing of wet materials is the mathematical modeling of the shape of the contact curves of the rollers and the regularities of the distribution of contact stresses along the contact curves of the rollers. The article deals with the basic contact problems of the theory of roller squeezing of leather. A formula was derived that describes the model of friction stresses. It was revealed that the shapes of the roll contact curves of the two-roll module do not depend on the change in the moisture content of hides in the contact zone of the rollers. It was established that the model of friction stresses in a two-roll module for pressing leather does not depend on the inclination of the material layer fed to the center line.

INTRODUCTION

The main working body of roller machines for squeezing out leather semi-finished products is a pair of working rollers with an elastic coating (a cloth). A pair of working rollers (a roll pair) and a layer of semi-finished leather after tanning (a layer of leather) together create a two-roll module for pressing leather.

The analysis of publications devoted to the study of hydraulic problems of roller pressing of wet materials [1-6] showed that the existing patterns of changes in the filtration rate in the pressing zone were obtained with the introduction of models of roller equipment and materials that do not correspond to the real contact phenomena of roller pressing of wet materials.

In [7-17], the equations of roll contact curves and patterns of contact stress distribution in two-roll modules are analytically described, without considering the moisture filtration from the wet material during the squeezing process.

This study is devoted to modeling the contact curves of two-roll modules of leather pressing. The filtration of moisture from the leather semi-finished product during the pressing process and the deformation properties of the semi-finished leather product are taken into account.

MATERIALS AND METHODS

The analysis of the roller machines for hides pressing [5] showed that, basically, the two-roll modules for pressing leather have a symmetrical form.

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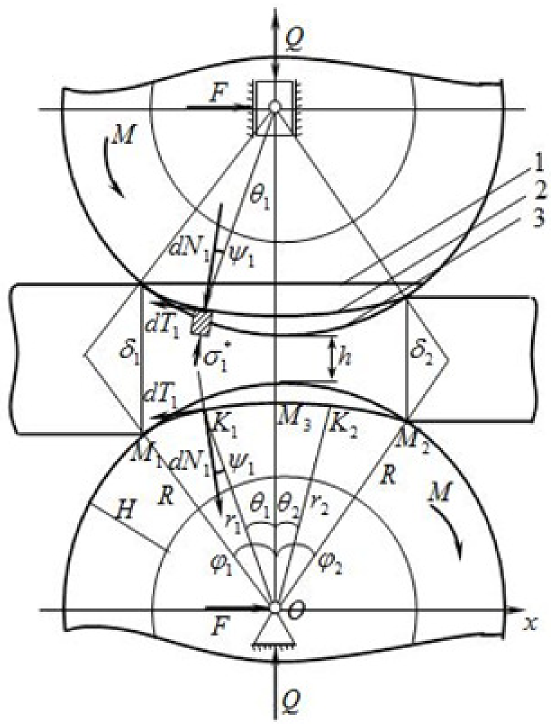

FIGURE 1. Scheme of interaction
in a two-roll module for pressing leather

Consider a two-roll module for pressing leather, in which rollers of radii R have an elastic cloth covering of a thickness of H, the leather layer thickness is δ_1, the distance between the rolls is h, both rolls are driven (Fig. 1).

Since the two-roll module under consideration is symmetric, we will investigate the contact interaction of leather with any roller, for example, with the lower roller. The roll contact curve consists of compression and recovery zones (Fig. 1). The points of the roll compression zone are determined by the polar coordinates r_1 and θ_1, and the points of the recovery zone are determined by r_2 and θ_2, where

$$-\varphi_1 \le \theta_1 \le 0, \quad 0 \le \theta_2 \le \varphi_2, \tag{1}$$

where φ_1, φ_2 are the contact angles.

From Figure 1 it follows that

$$2R\cos\varphi_1 + \delta_1 = 2R + h.$$

Hence, we determine

$$\varphi_1 = \arccos\sqrt{\frac{2R + h - \delta_1}{2R}}. \tag{2}$$

Likewise (2), we obtain

$$\varphi_2 = \arccos\sqrt{\frac{2R + h - \delta_2}{2R}}, \tag{3}$$

where δ_2 is the finite thickness of the leather layer.

According to [18], the deformation properties of leather (of the semi-finished leather product after chrome tanning) under compression and recovery are described by power-law dependences:

"stress – relative deformation" dependence:

$$\sigma = 43.39\varepsilon_c^{\ 3,23}; \qquad \sigma = 85.93\varepsilon_{\text{в}}^{\ 3,03}, \tag{4}$$

"humidity – relative deformation" dependence:

$$W = 1.13\varepsilon_c^{\ 0,2}; \qquad W = 1.24\varepsilon_{\text{в}}^{\ 0,24}, \tag{5}$$

where ε_c, $\varepsilon_{\text{в}}$ are the relative deformation of compression and recovery.

According to expressions (4) and (5), the deformation properties of leather under compression are described by the following dependencies

$$\sigma_1^* = \alpha_1 \varepsilon_1^{*s_1}, \quad W_1^* = W_{\text{н}}^* - \beta_1 \varepsilon_1^{*l_1}, \tag{6}$$

$W_{\text{н}}^*$ is the initial moisture of hides; $\alpha_1 = 43.39$; $\beta_1 = 1.13$; $s_1 = 3.23$; $l_1 = 0.2$.

From dependencies (6), we have

$$\frac{\sigma_1^*}{W_{\text{н}}^* - W_1^*} = \frac{\alpha_1}{\beta_1} \varepsilon_1^{*(s_1 - l_1)},$$

or

$$\sigma_1^* = B_1 \varepsilon_1^{*m_1^*} (W_{\text{н}}^* - W_1^*), \tag{7}$$

where $B_1 = \dfrac{\alpha_1}{\beta_1}$, $m_1 = s_1 - l_1$ ($B_1 = 37.51$; $m_1 = 3.03$).

From the analysis of the research results [19], it follows that the deformation of the roller coating (a cloth) under compression and recovery can be described by dependencies of the form

$$\sigma_j = A_j \varepsilon_j^{m_j} (W_j - W_{j\text{н}}), \quad j = 1,2, \tag{8}$$

where $W_{jн}$ is the moisture content of the cloth at the initial point of the $j-$th zone of the roll contact curve.

At each point of the contact curve, deformation of leather and roller coating occurs along the normal line $n-n$. Therefore, at each point of the compression zone M_1M_3, the stress equivalence is fulfilled (according to Newton's law), that is, $\dfrac{\sigma_1}{\cos\psi_1} = \dfrac{\sigma_1^*}{\cos\psi_1}$, where $\psi_1 -$ is the angle between the radius r_1 and the line $n-n$.

Hence

$$\sigma_1 = \sigma_1^*.$$

Therefore, it can be considered that the deformation of leather and roller coating in the two-roll module occurs in the radial direction to the axis of the rollers.

Then, according to expressions (7) and (8), we have

$$A_1 \varepsilon_1^{m_1} (W_1 - W_{1\text{н}}) = B_1 \varepsilon_1^{*m_1^*} (W_н^* - W_1^*), \tag{9}$$

where $\varepsilon_1, A_1, m_1, \varepsilon_1^*, B_1, m_1^*$ are the deformation, the strain and hardening coefficients of the cloth and leather under compression, respectively.

During the squeezing process, the moisture removed from the hides to point K_1 is transferred to the roller coating along the contact curve M_1M_2. The amount of moisture removed from hides to the point K_1 – $W_{1уд}^* = W_н^* - W_1^*$ is equal to the amount of moisture that has passed to the roller coating up to the point $W_{1п} = (W_1 - W_{1н})$.

Thus, for each point of the compression zone M_1M_3 of the lower roller, the following equivalence holds

$$(W_1 - W_{1н}) = (W_н^* - W_1^*). \tag{10}$$

Then from equivalence (9), we have

$$A_1\varepsilon_1^{m_1} = B_1\varepsilon_1^{*m_1^*}. \tag{11}$$

Differentiating both sides of equation (11), we determine

$$m_1 A_1 \varepsilon_1^{m_1-1} d\varepsilon_1 = m_1^* B_1 \varepsilon_1^{*m_1^*-1} d\varepsilon_1^*.$$

Hence

$$\frac{\varepsilon_1}{\varepsilon_1^*} = \frac{m_1}{m_1^*}\cdot\frac{d\varepsilon_1}{d\varepsilon_1^*} = \frac{m_1}{m_1^*}\lambda_1, \tag{12}$$

where $\lambda_1 = \dfrac{d\varepsilon_1}{d\varepsilon_1^*}$ is the ratio of the relative deformations of the elastic roll cover and the leather layer under compression.

According to Fig. 1, the relative deformations of the contacting bodies have the form

$$\varepsilon_1 = \frac{R - r_1,}{H} \qquad \varepsilon_1^* = \frac{r_1 - R\dfrac{\cos\varphi_1}{\cos\theta_1}}{\dfrac{\delta_1}{2\cos\varphi_1}}. \tag{13}$$

From equation (12), taking into account expression (13), we determine

$$R - r_1 = k_1\lambda_1\left(r_1 - R\frac{\cos\varphi_1}{\cos\theta_1}\right), \tag{14}$$

where $k_1 = \dfrac{2m_1 H\cos\varphi_1}{m_1^*\delta_1}$.

Having solved equation (14) with respect to r_1, we find the equation of the contact curve of the compression zone

$$r_1 = \frac{R}{1 + k_1\lambda_1}\left(1 + k_1\lambda_1\frac{\cos\varphi_1}{\cos\theta_1}\right). \tag{15}$$

Likewise formula (15), we find the equation for the contact curve of the recovery zone

$$r_2 = \frac{R}{1+k_2\lambda_2}\left(1+k_2\lambda_2\frac{\cos\varphi_2}{\cos\theta_2}\right), \tag{16}$$

where $k_2 = \dfrac{m_2 H\cos\varphi_2}{m_2^*\delta_2}$; λ_2 is the ratio of the rates of relative deformations of the elastic roll coating and the leather layer under recovery.

Generalizing Eqs. (15) and (16), we obtain

$$\begin{cases} r_1 = \dfrac{R}{1+k_1\lambda_1}\left(1+k_1\lambda_1\dfrac{\cos\varphi_1}{\cos\theta_1}\right), & -\varphi_1 \le \theta_1 \le 0, \\ r_2 = \dfrac{R}{1+k_2\lambda_2}\left(1+k_2\lambda_2\dfrac{\cos\varphi_2}{\cos\theta_2}\right), & 0 \le \theta_2 \le \varphi_2. \end{cases} \tag{17}$$

The system of equations (17) describes the roll contact curves of the two-roll module for pressing leather.

If in a two-roll module, the rollers have a non-deformable coating, then $\lambda_j = 0$. It follows from system (17) that $r = R,\ -\varphi \le \theta \le \varphi.$

If in the two-roll module the leather layer is non-deformable, then $\lambda_j = \infty$. It follows from system (17) that $r = \dfrac{R\cos\varphi}{\cos\theta},\ -\varphi \le \theta \le \varphi.$

Thus, any graph of the roll contact curve of the two-roll module of the roller squeezing of leather (curve 2) lies between the graphs of curves $r = \dfrac{R\cos\varphi}{\cos\theta}$ (curve 1) and $r = R$ (curve 3) (Fig. 1).

From the system (17), it was found that the shapes of the roll contact curves of the two-roll module do not depend on the change in the moisture content of hides in the contact zone of the rollers.

Let us analyze the stress state in a two-roll module for leather squeezing.

In the steady-state process of interaction, the roll is acted upon by the force of the pressing devices Q, the horizontal response of the roll supports F, the moment of resistance forces M, elementary forces of normal pressure and friction acting along the entire roll contact curve. Elementary forces in compression (N_1, T_1) and recovery (N_2, T_2) zones will be presented separately.

Considering the roll in balance under the action of the applied forces, we obtain

$$\begin{cases} dN_{1x} + dT_{1x} + dF_{1x} + dQ_{1x} = 0, \\ dN_{1y} + dT_{1y} + dF_{1y} + dQ_{1y} = 0, \end{cases} \tag{18}$$

where $N_{1x},\ N_{1y},\ T_{1x},\ T_{1y}$ – are the projections of the main normal and shear forces of the compression zone on the x and y axes.

From the diagram of the forces of the compression zone (Fig. 1), we have

$$\begin{gathered} dN_{1x} = dN_1\sin(\theta_1-\psi_1),\ \ dN_{1y} = -dN_1\cos(\theta_1-\psi_1),\ \ dT_{1x} = -dT_1\cos(\theta_1-\psi_1), \\ dT_{1y} = -dT_1\cos(\theta_1-\psi_1),\ \ dF_{1x} = dF_1,\ \ dF_{1y} = 0,\ \ dQ_{1x} = 0,\ \ dQ_{1y} = dQ_1, \end{gathered} \tag{19}$$

where ψ_1 – is the angle between force dN_1 and radius r_1.

From system (18), considering Eq. (19), for the compression zone, we have

$$\frac{dF_1}{dQ_1} = \frac{dT_1\cos(\theta_1-\psi_1) - dN_1\sin(\theta_1-\varphi_1)}{dT_1\sin(\theta_1-\psi_1) + dN_1\cos(\theta_1-\psi_1)}. \tag{20}$$

Since we are considering a steady-state process, we can assume that $\frac{F_1}{Q_1} = C_1$, where C_1 is a constant value.

Hence, $\frac{dF_1}{dQ_1} = \frac{F_1}{Q_1} = C_1$. Then from equation (20) we obtain

$$\frac{dT_1}{dN_1} = \frac{\sin(\theta_1 - \psi_1) + C_1 \cos(\theta_1 - \varphi_1)}{\cos(\theta_1 - \psi_1) - C_1 \sin(\theta_1 - \psi_1)}. \tag{21}$$

The following relations relate elementary forces to contact stresses [6]:

$$dN_1 = n_1 \sqrt{r_1^2 + r_1'^2}\, d\theta_1, \qquad dT_1 = t_1 \sqrt{r_1^2 + r_1'^2}\, d\theta_1, \tag{22}$$

where $n_1 = n_1(\theta_1)$, $t_1 = t_1(\theta_1)$, are the normal and shear stresses distributed over the compression zone of the roll contact curve, respectively.

We substitute expressions (22) into equation (21) obtain the dependences connecting shear and normal stresses at the points of the compression zone of the roll contact curve

$$t_1 = \frac{\sin(\theta_1 - \psi_1) + C_1 \cos(\theta_1 - \varphi_1)}{\cos(\theta_1 - \psi_1) - C_1 \sin(\theta_1 - \psi_1)} n_1. \tag{23}$$

Formula (23), which connects shear and normal stresses, is also called a friction stress model.

The friction stress model of the recovery zone is obtained likewise

$$t_2 = \frac{\sin(\theta_2 - \psi_2) + C_2 \cos(\theta_2 - \varphi_2)}{\cos(\theta_2 - \psi_2) - C_2 \sin(\theta_2 - \psi_2)} n_2, \tag{24}$$

where $C_2 = \frac{F_2}{Q_2}$.

Note that at the point of the contact curve lying on the line of centers, the following boundary conditions are satisfied: $t_1(0) = t_2(0)$, $n_1(0) = n_2(0)$, $r_1(0) = r_2(0)$, $r_1'(0) = r_2'(0) = 0$.

These conditions lead to equivalence $C_1 = C_2$. Then, $C = C_1 = C_2 = \frac{F}{Q}$.

The coefficient $C = \frac{F}{Q}$ is called a dynamic factor of the roll [10].

Therefore, generalizing Eqs. (23) and (24), we obtain

$$\begin{cases} t_1 = \dfrac{\sin(\theta_1 - \psi_1) + C \cos(\theta_1 - \psi_1)}{\cos(\theta_1 - \psi_1) - C \sin(\theta_1 - \psi_1)} n_1, & -\varphi_1 \le \theta_1 \le 0, \\ t_2 = \dfrac{\sin(\theta_2 - \psi_2) + C \cos(\theta_2 - \psi_2)}{\cos(\theta_2 - \psi_2) - C \sin(\theta_2 - \psi_2)} n_2, & 0 \le \theta_2 \le \varphi_2. \end{cases} \tag{25}$$

The system of equations (25) determines the model of friction stresses in the two-roll module for pressing leather. It shows that the model of friction stresses in a two-roll module for pressing leather does not depend on the inclination of the material layer fed to the center line and on the inclination of the upper roller relative to the vertical line.

Assuming that $tg\,\xi = C$, we transform system of equation (25) to the form

$$\begin{cases} t_1 = tg(\theta_1 - \psi_1 + \xi) n_1, & -\varphi_1 \le \theta_1 \le 0, \\ t_2 = tg(\theta_2 - \psi_2 + \xi) n_2, & 0 \le \theta_2 \le \varphi_2, \end{cases} \tag{26}$$

The system of equations (26) defines, in a simplified form, the models of friction stresses in a two-roll module for pressing leather.

In the deformation zone under compression, we select an element of a length of dl_1 directed along the $n-n$ line since the deformation of the leather layer occurs in this direction (Fig. 1). Elementary normal dN_1 and shear dT_1 forces and the response of the cutoff parts of the leather layer act on the selected element of the leather layer from the side of the lower roller. We ignore the forces of inertia and gravity acting on the selected layer element due to their smallness.

The force components dN_1 and dT_1 in the $n-n$ direction are balanced by the force $\sigma_1^* dl_{11}$ (Fig. 1):

$$\sigma_1^* dl_1 - dN_1 \cos 0^o - dT_1 \sin 0^0 = 0$$

or

$$\sigma_1^* = n_1 , \tag{27}$$

where σ_1^* is the stress of the leather layer pressing in the $n-n$ direction.

Then, taking into account equations (6) and (7), we obtain

$$n_1 = \alpha_1 \varepsilon_1^{*s_j} . \tag{28}$$

It follows from equation (13) that

$$\varepsilon_1^* = \frac{2\cos\varphi_1}{\delta_1}\left(r_1 - R_1 \frac{\cos\varphi_1}{\cos\theta_1} \right).$$

or after substituting the expression for r_1 from equation (15), we obtain

$$\varepsilon_1^* = \frac{2R\cos\varphi_1}{\delta_1(1+k_1\lambda_1)}\left(1 - \frac{\cos\varphi_1}{\cos\theta_1} \right). \tag{29}$$

Substituting expression ε_1^* into equation (28), we find the distribution patterns of the normal stress in the compression zone of the roll contact curve

$$n_1 = A_1 \left(1 - \frac{\cos\varphi_1}{\cos\theta_1} \right)^{s_1} , \tag{30}$$

where $A_1 = \alpha_1 \left(\frac{2R\cos\varphi_1}{\delta_1(1+k_1\lambda_1)} \right)^{s_1}$.

The distribution patterns of normal stresses in the recovery zone of the roll contact curve are determined likewise

$$n_2 = A_2 \left(1 - \frac{\cos\varphi_2}{\cos\theta_2} \right)^{s_2} , \tag{31}$$

where $A_2 = \alpha_2 \left(\frac{2R\cos\varphi_2}{\delta_2(1+k_2\lambda_2)} \right)^{s_2}$.

From equations (30) and (31), taking into account the system of equations (26), we find the distribution patterns of shear stresses along the roll contact curve of the two-roll module for pressing leather.

They have the following form

$$t_1 = A_1 \left(1 - \frac{\cos\varphi_1}{\cos\theta_1} \right)^{s_1} tg(\theta_1 - \psi_1 + \xi), \tag{32}$$

$$t_2 = A_2 \left(1 - \frac{\cos\varphi_2}{\cos\theta_2} \right)^{s_1} tg(\theta_2 - \psi_2 + \xi). \tag{33}$$

To analyze the distribution pattern of shear stresses along the roll contact curve, the point at which the shear stress is zero, that is, the neutral point, is of particular importance.

Let the neutral point of the roll contact curve be determined by the angle $(-\varphi_3)$.

Then, from formula (26), it follows that the following equations are true for the drive roller

$$tg(-\varphi_3 - \psi_1(-\varphi_3) + \xi) = 0$$

or

$$tg(-\varphi_3 - \psi_1(-\varphi_3) + \xi) \approx tg(-\varphi_3 - \psi_1(-\varphi_3)) + tg\xi =$$

$$= -\frac{Q\varphi_3 - F(1+k_1\lambda_1\cos\varphi_1)}{Q(1+k_1\lambda_1\cos\varphi_1)} = 0.$$

Hence, we have

$$\varphi_3 = \frac{F(1+k_1\lambda_1\cos\varphi_1)}{Q}. \tag{34}$$

Thus, mathematical models of the distribution of contact stresses along the roll contact curve of a two-roll module for pressing leather were obtained.

RESULTS AND DISCUSSIONS

Analysis of the calculated data on the obtained models of the contact stresses distribution indicates that the normal and shear contact stresses are distributed unevenly over the roll contact surface:

- normal contact stresses vary from zero at the beginning and end of the roll contact zone to a maximum at a point lying on the center line;
- the tangential contact stresses change their signs at the neutral point, which in the driving roll is located at the inlet of the material layer into the contact zone of the rolls, and in the non-driven roll towards the exit (Fig. 2).

The findings are fully consistent with the results of theoretical and experimental studies [7, 8, 11,14].

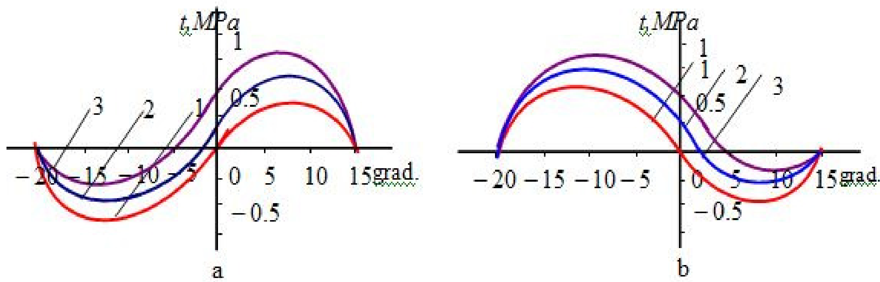

FIGURE. 2. Graphs of shear stress distribution in the driven roll (a) and in the non-driven roll (b): $1 - C = 0$; $2 - C = 0.05$; $3 - C = 0.1$.

The article deals with the main contact problems of the theory of roller pressing of leather.

Mathematical models of the shape of the roll contact curves and the distribution patterns of contact stresses along the roll contact curves were developed.

A formula was derived that describes the model of friction stresses.

It was revealed that the shapes of the roll contact curves of the two-roll module do not depend on the change in the moisture content of hides in the contact zone of the rollers.

It was established that the model of friction stresses in a two-roll module for pressing leather does not depend on the inclination of the material layer fed to the center line.

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